JEE/BITSAT PRACTICE TEST
Relations & Functions

Booklet Code A/B/C/D **Test Code : 2004**

Answer Key/Hints

1. If $f(x) = (a - x^n)^{1/n}$, then f (f (x)) =

(a) x (b) a-x (c) x^n (d) $x^{1/n}$

Sol. $f(x) = (a - y)^n)^{1/n} = y \therefore f(y) = (a - y^n)^{1/n} = \left[a - [(a - x^n)^{\frac{1}{n}}]^n\right]^{\frac{1}{n}} = \left[a - (a - x^n)\right]^{\frac{1}{n}} = (x^n)^{1/n} = x$

2. If f(x) = $\frac{3x+2}{5x-3}$, then

(a) $f^{-1}(x) = f(x)$ (b) $f^{-1}(x) = -f(x)$ (c) f (f(x)) = -x (d) $f^{-1}(x) = -\frac{1}{19}f(x)$

Sol. Let y = f(x) = $\frac{3x+2}{5x-3}$ $\therefore$ 5xy − 3y = 3x+2 $\Rightarrow$ x(5y − 3) = 2 + 3y

$\Rightarrow x = \frac{3y+2}{5y-3} \Rightarrow f^{-1}(y) = \frac{3y+2}{5y-3}$ $\Rightarrow$ $f^{-1}(x) = \frac{3x+2}{5x-3} = f(x)$

3. The range of the function

f(x) = cos [x], $-\frac{\pi}{4} < x < \frac{\pi}{4}$ where [x] denotes the greatest integer ≤ x is

(a) {0} (b){0, -1} (c){0, 1} (d) {1, cos 1}

Sol. When $-\frac{\pi}{4} < x < 0 \Rightarrow -1 < -\frac{\pi}{4} < x < 0 \Rightarrow$ [x] = -1 $\Rightarrow$ f(x) = cos (-1) = cos 1

when $0 \le x \le \frac{\pi}{4} < 1 \Rightarrow$ [x] = 0 $\Rightarrow$ f(x) = cos 0 = 1 $\therefore R_f = \{1, \cos 1\}$

4. If f(x) = cos (log x), then $f(x)\, f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$ has the value

(a) 1 (b) $\frac{1}{2}$ (c) -2 (d) 0

Sol. $f(x)\, f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$ = cos (log x) cos (log y) $-\frac{1}{2}\left[\cos\left(\log\left(\frac{x}{y}\right)\right) + \cos(\log(xy))\right]$

= cos (log x) cos (log y) $-\frac{1}{2}[\cos(\log x - \log y) + \cos(\log x + \log y)$ (Apply Cos C+Cos D)

= cos (log x) cos (log y) $-\frac{1}{2}[2\cos(\log x)\cos(\log y)] = 0$

5. The domain of definition of the function $y = 3e^{\sqrt{x^2-1}}\log(x-1)$ is

(a) $(1, \infty)$ (b) $[1, \infty)$ (c) R-{1} (d) None of these

Sol. $x^2 - 1 \ge 0$ and $x - 1 > 0 \Rightarrow |x| \ge 1$ and $x > 1 \Rightarrow x > 1 \Rightarrow D_f = (1, \infty)$

6. The range of the function $f(x) = \frac{x}{1+|x|}$ is

(a) [-1, 1] (b) R (c) (-1, 1) (d) R – {0}

Sol. When x ≥ 0 f(x) = $\frac{x}{1+x}$ $0 < f(x) < 1$ When x < 0, then |x| = -x ⇒f(x) = $\frac{x}{1-x}$

Since x < 0 ∴ -1 + x < x < 0 ⇒ -(1 – x) < x < 0 ⇒ -1 < $\frac{x}{1-x} < 0 \Rightarrow -1 < f(x) < 0$

Combining the two cases, -1 < f(x) < 1 ∴ R_f = (-1, 1)

7. The domain of the function f(x) = $\sin^{-1}\left(\log_3\left(\frac{x}{3}\right)\right)$ is

(a) [-1, 9] (b) [1, 9] (c) [-9, 1] (d) [-9, -1]

Sol. The function f is defined only if $-1 \le \log_3(x/3) \le 1$. This inequality is possible only if $3^{-1} \le x/3 \le 3$ $1/3 \le x/3 \le 3$ i.e., $1 \le x \le 9$.

8. The function $f(x) = \sec^{-1}\frac{x}{\sqrt{x-[x]}}$ is defines for

(a) all real x (b) R–{(-1, 1) ∪ Z} (c) R^+-(0, 1) (d) R^+-Z

Sol. $\sec^{-1}$ x is defined for x ≥ 1 or x ≤ -1
Also x –[x] ≠0 ∴ x is not an integer [∵ [x] = n and n – [n] = 0 for all integer n]
Hence domain of $\sec^{-1}\frac{x}{\sqrt{x-[x]}}$ =R-{(-1,1)UZ}where Z is the set of all integer

9. The function f : A → B defined by f(x) = $-x^2 + 6x - 8$ is a bijection, if

(a) $A = (-\infty, 3]$ and $B = (-\infty, 1]$ (b) $A = [-3, \infty)$ and $B = (-\infty, 1]$

(c) $A = (-\infty, 3]$ and $B = [1, \infty)$ (d) $A = [3, \infty)$ and $B = [1, \infty)$

10. Let f(x) = x^2 and g(x) = 2^x. Then the solution set of the equation fog(x)=gof (x) is

(a) R (b) {0} (c) {0, 2} (d) none of these

11 The domain of $\sin^{-1}\left(\frac{2x+1}{3}\right)$ is

(a) (-2, 1) (b) [-2, 1] (c) R (d) [-1. 1]

12 Two function f: R → R and g: R → R are defined as follows

f(x) = $\begin{cases} 0 & (x\ rational) \\ 1 & (x\ irrational) \end{cases}$ $g(x) = \begin{cases} -1 & (x\ rational) \\ 0 & (x\ irrational) \end{cases}$ then (gof)(e) + (fog) (π) =

(a) -1 (b) 0 (c) 1 (d) 2

Sol.(gof)(e) + (fog)(π) = g(f(e)) + f(g(π)= g(1) + f(0) = -1 + 0 = -1

13 If $e^{f(x)} = \frac{10+x}{10-x}, x \in (-10, 10)$ and $f(x) = kf\left(\frac{200x}{100+x^2}\right)$, then k =

(a) 0.5 (b) 0.6 (c) 0.7 (d) 0.8

Sol. $e^{f(x)} = \frac{10+x}{10-x} \Rightarrow f(x) = \log\left(\frac{10+x}{10-x}\right) \therefore f\left(\frac{200x}{100+x^2}\right) = \log\left(\frac{10+\frac{200x}{100+x^2}}{10-\frac{200x}{100+x^2}}\right)$

$= \log\left(\frac{1000+200x+10x^2}{1000-200x^2+10x^2}\right) = \log\left(\frac{10+x}{10-x}\right)^2 = 2\log\left(\frac{10+x}{10-x}\right) = 2f(x)$ by given by condition

f(x) = K.2f(x) ⇒ K = $\frac{1}{2} = 0.5$

14 If f(x) = $(25 - x^4)^{1/4}$ for 0 < x < $\sqrt{5}$, then $f\left(f\left(\frac{1}{2}\right)\right) =$

(a) 2^{-4} (b) 2^{-3} (c) 2^{-2} (d) 2^{-1}

Sol. $f\left(\frac{1}{2}\right) = \left(25 - \frac{1}{24}\right)^{\frac{1}{4}} = \left(25 - \frac{1}{16}\right)^{\frac{1}{4}} = \left(\frac{399}{16}\right)^{\frac{1}{4}}$ $f\left(f\left(\frac{1}{2}\right)\right) = \left(25 - \frac{399}{16}\right)^{\frac{1}{4}} = \left(\frac{1}{16}\right)^{\frac{1}{4}}$ **=** $\left(\frac{1}{2^4}\right)^{\frac{1}{4}} = \frac{1}{8} = 2^{-1}$

15 If f(x) = $\log\frac{1+x}{1-x}$, then

(a) f(x) is even (b) $f(x_1). f(x_2) = f(x_1 + x_2)$

(c) $\frac{f(x_1)}{f(x_2)} = f(x_1 - x_2)$ (d) f(x) is odd

16. The period of the function $f(x) = \operatorname{cosec}^2 3x + \cos 4x$ is

(a) $\pi/3$ (b) $\pi/4$ (c) $\pi/6$ (d) π

17. The period of the function $y = \sin\frac{2t+3}{6\pi}$ is

(a) 2π (b) 6π (c) $6\pi^2$ (d) 4π

Sol. Period is equal to $2\pi/(2/6\pi) = 6\pi^2$

18. The domain of $y = \sqrt{\log_{10}\left(\frac{5x-x^2}{4}\right)}$ is

(a) [0, 5] (b) [1, 4] (c) [-1, 2] (d) [1, 2]

Sol. We must have $\log_{10}\left(\frac{5x-x^2}{4}\right) \ge 0 \Leftrightarrow \frac{5x-x^2}{4} \ge 1 \Leftrightarrow (x-4)(x-1) \le 0 \Leftrightarrow x \in [1,4]$.

19. The domain of $y = \sin^{-1}\frac{x-3}{2} - \log_{10}(4-x)$ is

(a) $(-\infty, 4)$ (b) [1, 4) (c) $(-\infty, 3)$ (d) $(-\infty, 4)$

Sol. The $\sin^{-1}(x-3)/2$ is defined if $-1 \le (x-3)/2 \le 1$ and $\log_{10}(4-x)$ is defined if x<4. So we must have $1 \le x \le 5$ and x<4. Thus the domain is [1,4).

20. The period of $f(x)=\frac{1}{2}\left\{\frac{|\sin x|}{\cos x}+\frac{|\cos x|}{\sin x}\right\}$ is

(a) 2π (b) π (c) $\pi/2$ (d) $\pi/4$

Sol. $\because$ Periods of $|\sin x|, \cos x, |\cos x|, \sin x$ are $\pi, 2\pi, \pi, 2\pi$

respectively. LCM of all periods= 2π $\therefore$ Period of f(x) is 2π

21. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is

(a) {1, 2, 3, 4} (b) {1, 2, 3, 4, 5, 6} (c) {1, 2, 3} (d) {1, 2, 3, 4, 5}

Sol. $7-x \ge 1, x-3 \ge 0$ and $7-x \ge x-3 \Rightarrow$ $x \le 6, x \ge 3, x \le 5$.

Thus $3 \le x \le 5$ $\therefore$ $\text{Range} = \{4P_0, 3P_1, 2P_2\} = \{1, 3, 2\}$

22. The domain of the function f (x) = $\frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

(a) [1, 2] (b) [2, 3] (c) [1, 3] (d) [1, 2)

Sol. $-1 \le x-3 \le 1$ and $9-x^2 > 0 \Rightarrow$ $2 \le x \le 4$ and $-3 < x < 3$. So domain of f is [2, 3).

23. The period of the function f (x) **=** $\cos^2 3x$ + tan 4x is

(a) $\pi/3$ (b) $\pi/4$ (c) $\pi/6$ (d) π

Sol. f (x) = (1/2) (1 + cos 6x) + tan 4x. The period of cos 6x is $2\pi/6 = \pi/3$ and the period of tan4x is $\pi/4$. Hence the period of f is 1.c.m. of $\pi/3$ and $\pi/4 = \pi$.

24. Let f : [2, ∞) → X be defined by $f(x) = 4x - x^2$. Then, f is invertible if X =

(a) [2, ∞) (b) (-∞, 2] (c) (-∞, 4] (d) [4, ∞)

25. If f (x) = ax + b and g (x) = cx + d, then f(g(x)) = g(f(x)) for all $x \in R$ if and only if

(a) f(a) = g(c) (b) f(b) = g(b) (c) f (d) = g (b) (d) f(c) = g(a)

26. Let R={(3,3),(6,6), (9,9),(12, 12),(6, 12),(3,9),(3,12), (3,6)} be a relation on the set A = {3, 6, 9, 12}. The relation is

(a) An equivalence relation. (b) Reflexive and symmetric only.
(c) Reflexive and transitive only (d) Reflexive only.

Sol. R is reflexive as (3, 3), (6, 6), (9, 9), (12, 12), $\in$ R. R is not symmetric as (6,12) $\in$ R but (12, 6) $\notin$ R. R is transitive as the only pair which needs verification is (3, 6) and (6, 12) $\in R \Rightarrow$ (3, 12) $\in$ R.

27. If $g(x)=1+\sqrt{x}$ and $f[g(x)] = 3+2\sqrt{x}+x$, then f(x)=

(a) $1+2x^2$ (b) $2+x^2$ (c) $1+x$ (d) 2+x

Sol. We have, $g(x)=1+\sqrt{x}$ and $f[g(x)]=3+2\sqrt{x}+x$ ...(1)

Also, $f[g(x)]=f(1+\sqrt{x})$ ...(2)

From (1) and (2), we get $f(1+\sqrt{x})=3+2\sqrt{x}+x$ Let $1+\sqrt{x}=y$ or $x=(y-1)^2$

$\therefore f(y)=3+2(y-1)+(y-1)^2=3+2y-2+y^2-2y+1=2+y^2$ $\therefore$ $f(x)=2+x^2$

28. The domain of the function $f(x)=\log_{10}\frac{x-5}{x^2-10x+24}-3\sqrt{x+5}$ is

(a) $(-5,\infty)$ (b) $(5,\infty)$ (c) $(2,5)\cup(5,\infty)$ (d) $(4,5)\cup(6,\infty)$

29. The function f (x) = $\frac{x}{e^x-1}+\frac{x}{2}+1$ is

(a) even (b) periodic (c) odd (d) neither even nor odd

Sol. $f(x)=\frac{x}{e^x-1}+\frac{x}{2}+1$ changing $x\to -x$;

$f(-x)=\frac{-x}{e^{-x}-1}-\frac{x}{2}+1 \Rightarrow f(-x)=\frac{-x\,e^x}{1-e^x}-\frac{x}{2}+1$

$\Rightarrow f(-x)=\frac{+x(e^x)}{e^x-1}-\frac{x}{2}+1 \Rightarrow f(-x)=\frac{x(e^x-1+1)}{e^x-1}-\frac{x}{2}+1 \Rightarrow f(-x)=x\left[1+\frac{1}{e^x-1}\right]-\frac{x}{2}+1$

$\Rightarrow f(-x)=x+\frac{x}{e^x-1}-\frac{x}{2}+1 \Rightarrow f(-x)=\frac{x}{2}+\frac{x}{e^x-1}+1$; $f(-x)=f(x)$ $\therefore$ even function.

30. Let f(x) = $\frac{1-x}{1+x}$. Then fof (cos x) is equal to

(a) cos 2x (b) cos x (c) tan2x (d) tan x

31. Let X = Y = **R** $\{1\}$. The function $f:X\to Y$ defined by f (x) = $\frac{x+2}{x-1}$ is

(a) One-one but not onto (b) Onto but not one-one
(c) neither one-one nor onto (d) One-one and onto

32. If $g(x) = x^2 + x - 2$ and $\frac{1}{2}$ gof (x) $=2x^2 -5x+ 2$, then f(x) is equal to

(a) 2x – 3 (b) 2x + 3 (c) $2x^2 + 3x + 1$ (d) $2x^2 - 3x - 1$

33. Let A be a finite set containing n distinct elements. The number of one-one functions that can be defined from A to A is

(a) 2^n (b) n^n (c) 2^{n^2} (d) n!

34. Let $f(x)=\frac{\sin 2nx}{1+\cos^2 nx}$, $n\in N$ has $\frac{\pi}{6}$ as its fundamental period, then n is equal to

(a) 2 (b) 4 (c) 6 (d) 8

35. Let f(x) = [x] and g(x) = x – [x], then which of the following functions is the zero function?

(a) (f+g)(x) (b) (fg)(x) (c) (f – g)(x) (d) (fog)(x)

36. The inverse of the function f : R →{x ∈ R: x < 1} given by f(x) = $\frac{e^x - e^{-x}}{e^x + e^{-x}}$, is

(a) $\frac{1}{2}\log\frac{1+x}{1-x}$ (b) $\frac{1}{2}\log\frac{2+x}{2-x}$ (c) $\frac{1}{2}\log\frac{1-x}{1+x}$ (d) $\frac{1}{2}\log\frac{2-x}{2+x}$

37. If the function f : R → A given by f(x) = $\frac{x^2}{x^2+1}$ is a surjection, then A is

(a) R (b) [0, 1] (c) (0, 1] (d) [0, 1)

38. Let A = {2, 3, 4, 5, …., 17, 18}. Let '□' be the equivalence relation on A × A, Cartesian product of A with itself, defined by (a, b) □ (c, d) iff ad = bc. Then, the number of ordered pairs of the equivalence class of (3, 2) is

(a) 4 (b) 5 (c) 6 (d) 7

39. A relation ϕ from C to R is defined by x ϕ y ⇔ |x| = y. Which one is correct?

(a) (2 + 3i) ϕ 13 (b) 3 ϕ (-3) (c) (1 + i) ϕ 2 (d) i ϕ 1

40. Let R be a relation on the set N of natural numbers defined by n R m iff n divides m. Then, R is

(a) reflexive and symmetric (b) transitive and symmetric

(c) equivalence (d) reflexive, transitive but not symmetric